ETH-Summer School 2004

Rate-distortion optimized tree structured compression algorithms for piecewise smooth images

Joint work with R. Shukla, P. L. Dragotti, and M. N. Do

1. Introduction

Long standind tradition of tree-structured approaches in

- signal processing and communications: binary trees
- image processing and computer vision: quadtrees
- video and medical imaging: octrees

Reason:

- computational complexity
- tractability
- design
- successive refinement (or divide and conquer)

Some examples

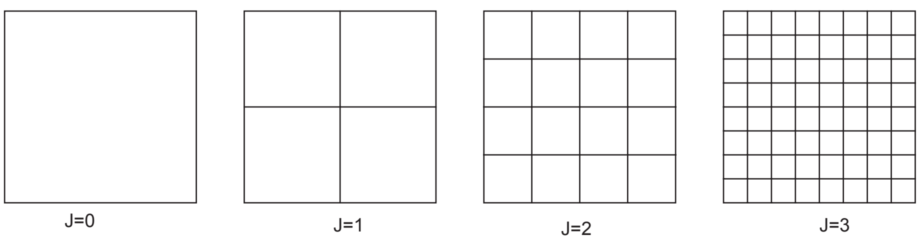
- split&merge image compressionm [Leonardi et al]
- binary space partitioning trees [Radha et al]
- Non-linear tilings [Cohen, Mattei]: adaptive segmentation
- NP-completeness of optimal tiling

Note: no basesor frames here!

Reminder

Tree structured algorithms

- quadtrees



Tree growing:

- greedy => suboptimal
- less complex

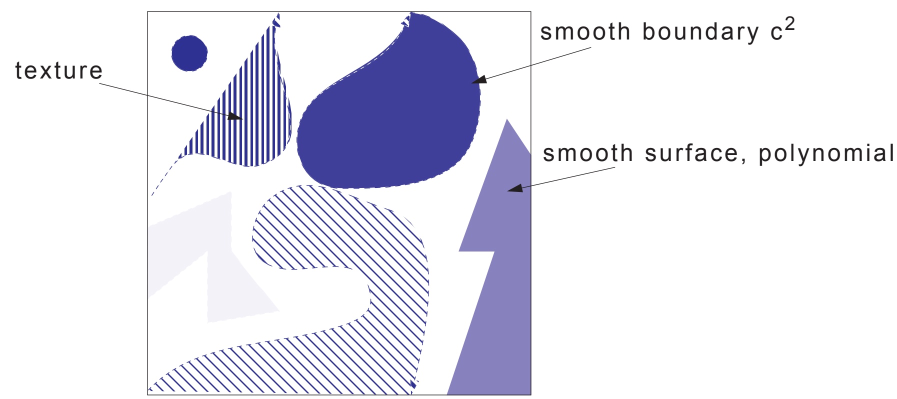
Tree pruning:

- costly
- under certain constraints: optimal
- Lagrangian optimization

2. Non-Separable Constructions Based on Quadtrees

Going to two dimensions requires non-separable bases

Objects in two dimensions we are interested in

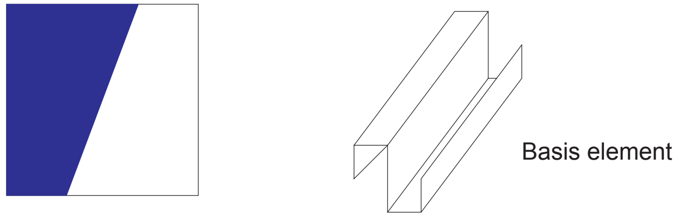


- textures: $D(R) = C_0 \cdot 2^{-2R}$ per pixel

  smooth surfaces: $D(R) = C_1 \cdot 2^{-2R}$ per object!

Compression of non-separable objects

Objects we know how to compress....



Approximation

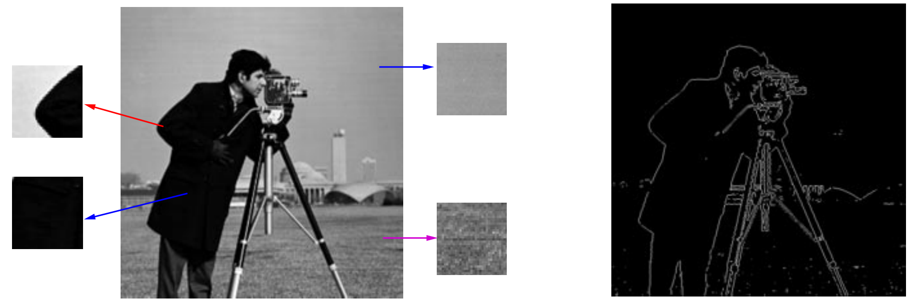
- Wavelets $E_M \sim M^{-1}$
- Ridgelets $E_M \sim 2^{-M}$

Rate/distortion

- Oracle $D(R) = C \cdot 2^{-2R}$
- Wavelets....poor
- Ridgelets....suboptimal
- adaptive schemes: close to oracle
- fixed basis: under investigation

Motivation: Natural images exhibit structure

Natural images represent a special class of 2-D functions.



- Dominant image structures:
  1. Smooth regions: Surface (2-D) regularity.
  2. Smooth edge contours: Geometrical (1-D) regularity (Perceptual).
- Image processing algorithms require efficient modeling/exploitation of both type of regularity. In particular, compression and denoising.

Idea

- tree and quadtree algorithms popular, many pruning algorithms optimality proofs for wedgelets [Donoho:99]

Here: new pruning and joining algorithm

Binary Tree Algorithms for 1-D piecewise polynomials

Prune Binary Tree (Parent Children Pruning) [~ Wavelet packets]

Step 1: Initialization:

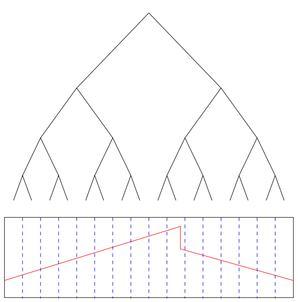
(a) Segment the signal using a binary tree.

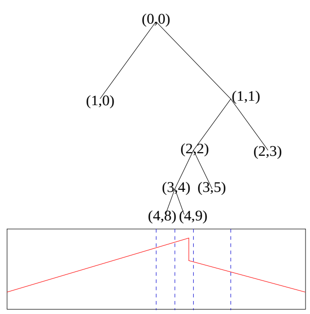
(b) Approximate each node by its best polynomial

(c) Generate R-D curve for each node of the tree.

Step 2: Prune the tree to minimize the Lagrangian cost $L(\lambda) = D + \lambda R$.

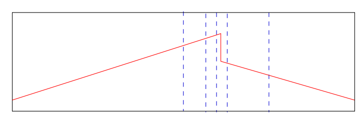
Step 3: Search $\lambda*$ for a desired bit budget $R*$.



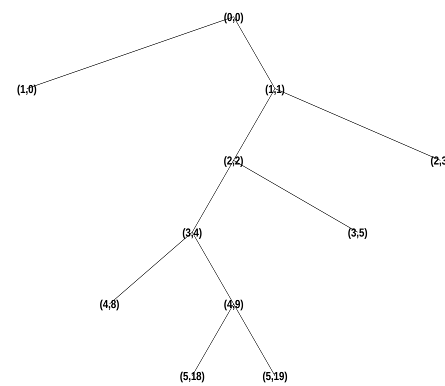


Prune Binary Tree

(Parent Children Pruning)

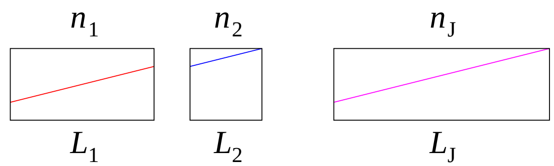


Prune binary tree segmentation

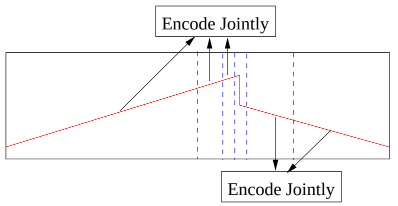


For piecewise polynomials: $D(R) \sim \sqrt{R} \cdot 2^{-c_1 \cdot \sqrt{R}}$

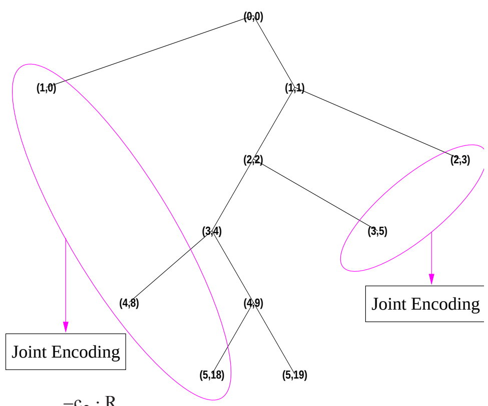
Neighbor Joint Coding Strategy



Encode jointly if: $L_J(\lambda) \le L_1(\lambda) + L_2(\lambda)$, where $L(\lambda) = D + \lambda R$.

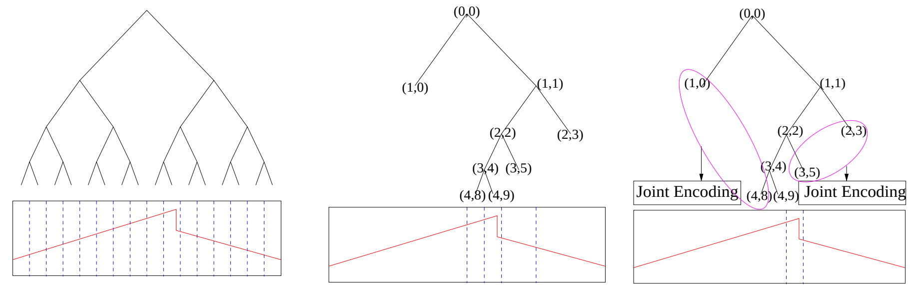


Binary tree segmentation along with joint encoding



For piecewise polynomials: $D(R) \sim 2^{-c_2 \cdot R}$

Binary Tree Segmentations (Recap)



(a) Full tree

$N_J \sim 2^J$

$D(R) \sim R^{-1}$

(b) Dyadic tree

$N_J \sim J$

$D(R) \sim \sqrt{R} \cdot 2^{-c_1 \cdot \sqrt{R}}$

(c) Prune-join tree

$N_J \sim J^0$

$D(R) \sim 2^{-c_2 R}$

Results: Rate-distortion optimal for piecewise polynomials

$D(R) = c_1 \cdot 2^{-(c_2 \cdot R)}$ **that is, like an oracle method (up to constants)**

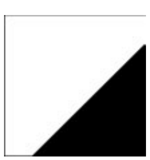
but at much lower computational complexity

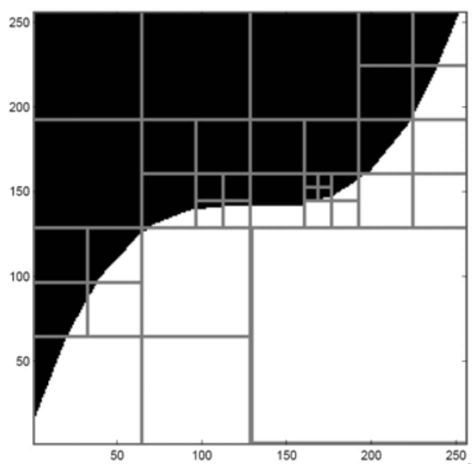
Extension to 2-D: Quadtree Algorithms

Algorithms are similar to the binary tree schemes in 1-D.

Prune Quadtree Algorithm:

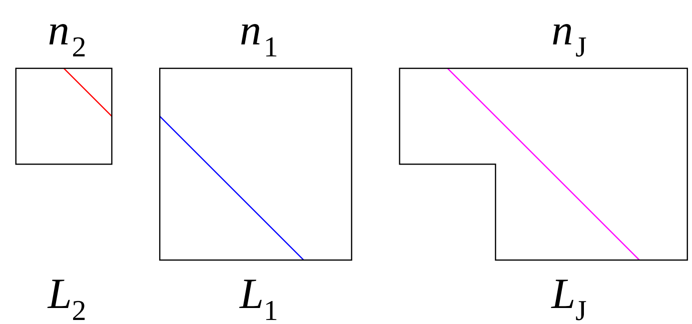
- Segment the image into dyadic squares.
- Code each block as an edge-tile with a linear discontinuity.
- Prune the tree to minimize the Lagrangian cost: $L(\lambda) = D + \lambda R$.





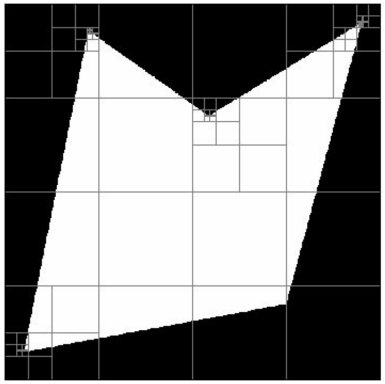
Prune-Join Quadtree Algorithm with Joint Coding

- Find the pruned tree using the Prune Quadtree Algorithm.
- Code neighbor segments with “similar” parameters jointly.



Encode jointly if: $L_J(\lambda) \le L_1(\lambda) + L_2(\lambda)$, where $L(\lambda) = D + \lambda R$.

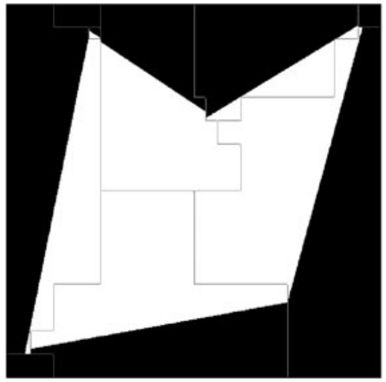
Polygonal Image Model



(a) Prune tree

$$N_J \sim J$$

$$D(R) \sim \sqrt{R} \cdot 2^{-c_1 \cdot \sqrt{R}}$$



(b) Prune-join tree

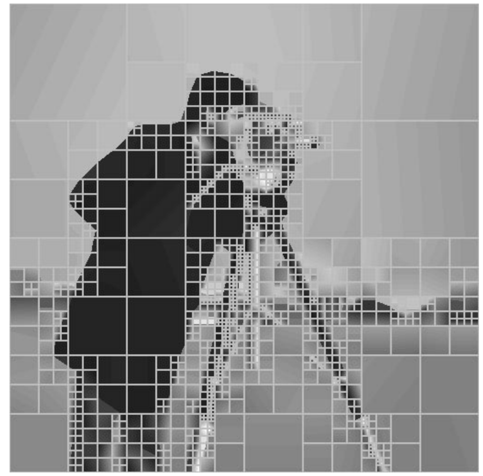
$$N_J \sim J^0$$

$$D(R) \sim 2^{-c_2 \cdot R}$$

Prune-join quadtree => Object based paradigm

The prune-join quadtree algorithmon an image

- polynomial fit to surface and to boundary on a quadtree
- rate-distortion optimal tree pruning and joining



quadtree with R(D) pruning

R(D) Joining of ''similar'' leaves

Note: careful R(D) optimization!

Upperbounds on R-D Behaviors

A. Boundary is piecewise polynomial:

- Oracle:

$$D(R) \sim 2^{-c \cdot R}$$

- The prune quadtree algorithm (independent coding):

$$D(R) \sim \sqrt{R} \cdot 2^{-c_1 \cdot \sqrt{R}}$$

- The prune-join quadtree algorithm with joint coding:

$$D(R) \sim 2^{-c_2 \cdot R}$$

B. Boundary is piecewise smooth:

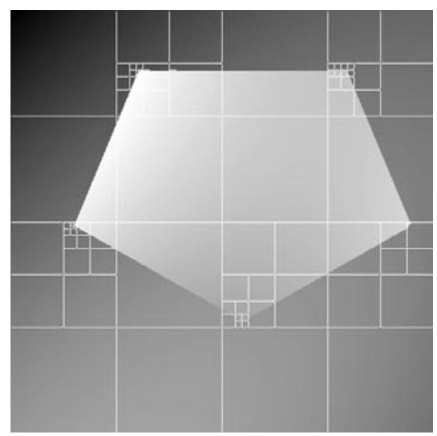
- The prune quadtree algorithm (code blocks independently) achieves the oracle performance (up to log factor):

$$D(R) \sim \left(\frac{\log R}{R}\right)^p$$

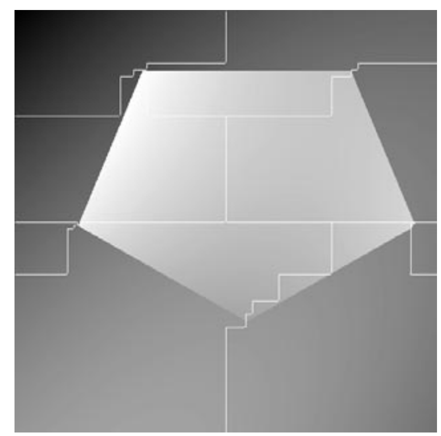
C. Computational complexity of quadtree algorithms:

$$O(N^2 \cdot \log N)$$

Simulation Results: Piecewise Polynomial Images

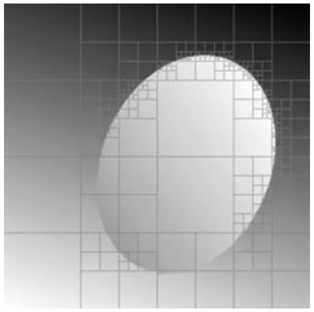


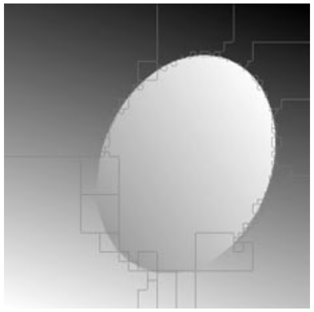
(a) Prune tree

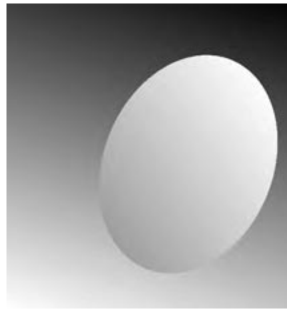


(b) Prune-join tree

Simulation Results: Piecewise Smooth Images







(a) Prune Tree

Rate=0.03 bpp

PSNR=44.43 dB

(b)Prune-join Tree

Rate=0.02 bpp

PSNR=44.24 dB

(c) JPEG2000

Rate=0.065 bpp

PSNR=43.81dB

Cameraman Image at 0.15 bpp

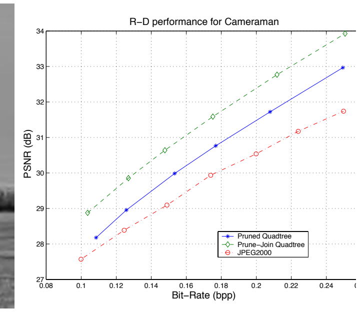
(a) Prune-join tree

(a) PSNR=30.68dB

(b) JPEG2000

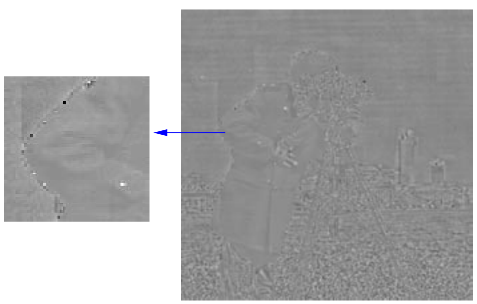
(b) PSNR=29.21 dB

(c) Quadtree vs JPEG2000

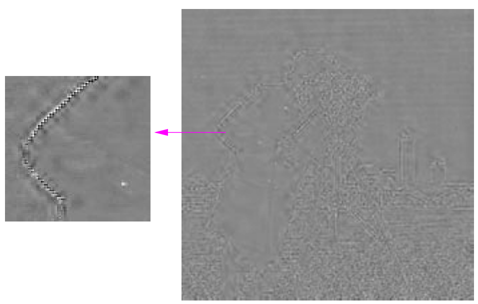


(c) R-D Performance

Residual cameraman image at 0.15 bpp



(a) Prune-join tree



(b) JPEG2000

Note:

- Standard residual coding cannot improve the overall R-D performance because residual image is neither smooth nor geometrically simple.

Lena image at 0.15 bpp

(a)Prune-join tree

(a) PSNR=30.86 dB

(b) JPEG2000

(b) PSNR=30.34 dB

Note

- Due to more texture and the small number of large smooth regions, the performance improvement is relatively small.

Behavior of tree algorithms on piecewise smooth fcts

ppf: piecewise polynomial functions

psf: piecewise smooth functions

Signal Class	Oracle Coder	Wavelet Coder	Prune tree Coder	Prune-join tree Coder
1-D PPF	2^{-cR}	$2^{-c_1\sqrt{R}}$	$2^{-c_2\sqrt{R}}$	2^{-c_3R}
2-D PPF	2^{-dR}	$\frac{\log R}{R}$	$2^{-c_4\sqrt{R}}$	2^{-c_5R}
1-D PSF	R^{-2p}	R^{-2p}	$\left(\frac{\log R}{R}\right)^{2p}$	$\left(\frac{\log R}{R}\right)^{2p}$
2-D PSF	R^{-p}	$\frac{\log R}{R}$	$\left(\frac{\log R}{R}\right)^{p}$	$\left(\frac{\log R}{R}\right)^{p}$

at most log penalty with polynomial complexity (and a bit more work gets rid of logs...)

Conclusions

Performance
- oracle like behavior for piecewise polynomials with polynomial boundaries
- similar behavior for piecewise smooth
- initial “practical” coder beats state of the art coder

Complexity
- $O(N^2)$

Other applications
- model based denoising

Publications

Thesis
- R.Shukla, Rate-distortion optimized geometrical image processing, PhD Thesis, EPFL, 2004.

Papers:

- A.Cohen, I.Daubechies, O.Gulieruz and M.Orchard, On the importance of combining wavelet-based non-linear approximation with coding strategies, IEEE Tr. on IT, 2002
- R. Shukla, P. L. Dragotti, M. N. Do and M. Vetterli, Rate-distortion optimized tree structured compression algorithms for piecewise smooth images, IEEE Transactions Image Processing, 2004.