JOINT ESTIMATION IN MRI USING HARMONIC RETRIEVAL METHODS

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ABSTRACT

One of the key factors affecting functional MRI image reconstruction is field inhomogeneity. It is desirable to estimate both the distortion-free MRI image and field map simultaneously, thus compensating for image distortions caused by the field inhomogeneity. To solve this problem, which is called Joint Estimation problem, we propose a new non-iterative approach using Harmonic Retrieval (HR) methods. This connection establishes an elegant framework to solve the Joint Estimation problem through a sequence of one dimensional HR problems in which efficient solutions are available. We also derive the condition on the smoothness of the field map in order for HR techniques to recover the image with high SNR. Experimental results with the proposed method show significant improvements in MRI image reconstruction compared to methods that do not correct for field inhomogeneity.

1. INTRODUCTION

Functional MRI (fMRI) is used to study localized brain function, both in examining healthy cognitive function and in clinical patient groups. Hence, fMRI reconstruction is important for assessing the risk of brain surgery or understanding the physiological basis for neurological disorders. Much research has been conducted on improving reconstruction performance. One of the main causes of severe distortions in MRI images, such as geometric distortions and blurring effects, is field inhomogeneity. To correct the distortions, standard field map estimation methods involves two steps: estimating magnetic field variation and then compensating for this variation during image reconstruction [1]. However, these methods implicitly assume that the local intrinsic signal does not change its amplitude or phase during signal acquisition, which is not correct. Another approach was suggested in [2] and [3] where the idea is to combine two steps together: reconstruct the undistorted image and field map simultaneously from the acquired data. This problem is called the Joint Estimation problem.

In [3] the Joint Estimation problem was solved using a nonlinear least-squares Conjugate Gradients (CG) method. In this work, we propose another non-iterative approach to the Joint Estimation problem using Harmonic Retrieval (HR) technique. We show that under certain approximation the Joint Estimation problem can be treated as a sequence of 1D HR problems in which efficient solutions are available. Advantages of this approach include the running time, robustness to incorrect local minima solutions of the CG method, and no prior information for field map is needed. In this paper, we concentrate on the case when k-space trajectory is EPI which is commonly used in the practice for functional imaging. Since the Joint Estimation problem estimates both the image and field map (2N unknowns), we need at least 2N data samples along the vertical axis (2N equations). Standard methods to estimate the field map also require acquiring two images in order to form this estimate [7]. We also point out a condition on the smoothness of the original field map and characterize the restoration quality as the function of field map. In the last part, we show experimental results that compare this approach with image reconstruction without field map and present conclusions and further work.

2. MATHEMATICAL MODEL OF THE PROBLEM

Given the signal during the readout $s(t)$ with specific k-space trajectory $\mathbf{k}(t)$, the Joint Estimation problem recovers the image $f(\mathbf{r})$ and field map $w(\mathbf{r})$ as described in the following equation:

$$s(t) = \int f(\mathbf{r}) e^{-i\omega(\mathbf{r})t} e^{-i2\pi(\mathbf{k}(t)\cdot\mathbf{r})} d\mathbf{r}. \tag{1}$$

Discretization of (1) leads to the following form [3]:

$$s(\mathbf{m}) = \sum_{\mathbf{n}} f(\mathbf{n}) e^{-i\omega(\mathbf{n})t(\mathbf{m})} e^{-i2\pi\mathbf{k}(t(\mathbf{m}))\cdot\mathbf{r}(\mathbf{n})} \Phi(\mathbf{k}(t(\mathbf{m}))), \tag{2}$$

where $\Phi(\mathbf{k}(t(\mathbf{m})))$ is voxel basis. For simplicity, from now on we will denote $s(\mathbf{m}) = \frac{s(t(\mathbf{m}))}{\Phi(\mathbf{k}(t(\mathbf{m})))}$.

Specifically, we assume an Echo-Planar Imaging (EPI) trajectory $k(t(\mathbf{m}))$ as described in Fig. 1.

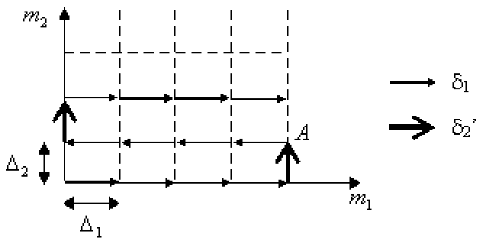


Fig. 1. EPI sampling trajectory: Δ_1, Δ_2 are sampling intervals in k-space and $\delta_1, \delta_2^{'}$ are time delays between samples as denoted by the arrows.

3. PROPOSED APPROACH

3.1. Problem analysis

We denote those k-space lines along which the data is acquired from left to the right as forward lines and lines along which the data is acquired from right to the left as backward lines.

Let δ_1, δ_2' be the time delays and Δ_1, Δ_2 be the space distances between two consecutive samples of the EPI trajectory along horizontal and vertical axes, respectively. The total number of samples along horizontal and vertical axes is M_1 and M_2.

To convert the problem (2) into a 2D problem, we express $t(\mathbf{m})$, $\mathbf{k}(\mathbf{m})$ and $\mathbf{r}(\mathbf{n})$ in a mathematical form. From Fig. 1, the timing delay for forward lines is:

$$t(\mathbf{m}) = m_1\delta_1 + m_2\delta_2 \tag{3}$$

and for backward lines is:

$$t(\mathbf{m}) = (M_1 - 1 - m_1)\delta_1 + m_2\delta_2, \tag{4}$$

where δ_2 is the distance from origin along EPI trajectory until point A (see Fig 1). In other words,

$$\delta_2 = \delta_1(M_1 - 1) + \delta_2'.$$

The function $k(\mathbf{m})$ has one single form for both forward and backward lines. For simplicity in derivation, we assume that k-space sampling starts from the origin as it is depicted in Fig. 1.

$$\mathbf{k}(\mathbf{m}) = \begin{pmatrix} m_1\Delta_1 \\ m_2\Delta_2 \end{pmatrix}. \tag{5}$$

The function $\mathbf{r}(\mathbf{n})$ can be expressed as:

$$\mathbf{r}(\mathbf{n}) = \frac{1}{N}\mathbf{n}, \tag{6}$$

where $\mathbf{n} = (n_1, n_2)$ is Cartesian coordinate system and N is the number of samples satisfying the Nyquist sampling condition (here, the size of the image is $N \times N$).

3.2. Derivation

We consider the forward case first and then show under some practical assumptions that the backward case will be exactly the same as the forward case.

1. *Forward lines*: substituting equations (3), (5) and (6) into (2), we obtain:

$$s(m_1, m_2) = \sum_{\mathbf{n}} f(\mathbf{n}) e^{-im_1(\omega(\mathbf{n})\delta_1 + \frac{2\pi}{N}\Delta_1 n_1)} \times e^{-im_2(\omega(\mathbf{n})\delta_2 + \frac{2\pi}{N}\Delta_2 n_2)}. \tag{7}$$

In practice, the time interval δ_1 between two samples along the horizontal k-space line is two orders of magnitude smaller than the time δ_2' needed for the gradient to change its value and go to the next line, therefore $\delta_1 \ll \delta_2$. Hence, we can assume that $w(\mathbf{n})\delta_1 \approx 0$. A similar approximation is widely used for EPI [1]. Setting $\Delta_1 = 1$ and taking inverse DFT both sides of equation along horizontal m_1 direction, we get:

$$\widehat{s}(n_1, m_2) = \sum_{n_2=0}^{N-1} f(n_1, n_2) e^{-i(\omega(\mathbf{n})\delta_2 + \frac{2\pi}{N} n_2) m_2}, \quad m_2 = 0, ..., M-1. \tag{8}$$

Comparing with the 1D HR problem

$$s(m) = \sum_{n=0}^{N-1} a_n e^{i\phi_n m}, \tag{9}$$
$$m = 0, ..., M-1 \quad \text{and} \quad M \geq 2N,$$

we see that (8) is a sequence of N 1D-HR problems where for a *fixed* n_1 we can assign $a_n = f(n_1, n_2)$ and $\phi_n = -\left(\frac{2\pi}{N} n_2 + \omega(\mathbf{n})\delta_2\right)$, which can be solved when $M \geq 2N$ (M data samples, $2N$ unknowns a_n and ϕ_n). This means the number of samples collected along the vertical k-space axis should be double the reconstructed size along same axis.

Two different methods exist to solve the HR problem: using an annihilating filter and a subspace-based approach [4]. A lot of variations of these methods have been proposed to enhance the overall performance in noiseless and noisy cases: Least Squares (LS) Prony, Total LS Prony, Pencil-based, IQML, Subspace, Forward-Backward methods [4], [5], [6].

2. *Backward lines*: by the same token, we get the following equation for backward case:

$$s(m_1, m_2) = \sum_{\mathbf{n}} f(\mathbf{n}) e^{-i(\omega(\mathbf{n})(M_1 - 1 - m_1)\delta_1 + \omega(\mathbf{n}) m_2 \delta_2)} \times e^{-i\frac{2\pi}{N}(m_1 n_1 \Delta_1 + m_2 n_2 \Delta_2)}. \tag{10}$$

Assuming again that $w(\mathbf{n})\delta_1 \approx 0$, we obtain the same problem formulation as equation (8) in forward case.

In the case where k-space is symmetric, the acquired data are $s(\tilde{k_1}, \tilde{k_2})$, where $-\frac{N}{2} \le \tilde{k_1} < \frac{N}{2}$ and $-\frac{N}{2} \le \tilde{k_2} < \frac{N}{2}$. Since $s(k_1, k_2)$ corresponds to the image $f(\mathbf{n})$, the shift in k-space, $s(k_1 - \frac{N}{2}, k_2 - \frac{N}{2})$ leads to a modulation term in the spatial domain: $\tilde{f}(\mathbf{n}) = f(\mathbf{n})e^{-j\pi n_1}e^{-j\pi n_2}$.

3.3. Algorithm for mapping to HR problem

Below is the summary of proposed algorithm (see Fig. 2):

1. Given acquisition data, take inverse DFT along m_1 axis, call it $\widehat{s}(n_1, m_2)$.

2. For each fixed value of n_1 solve following 1D HR problem to recover field map $\omega(\mathbf{n})$ and image $\tilde{f}(\mathbf{n})$:

$$\widehat{s}(n_1, m_2) = \sum_{n_2} \tilde{f}(n_1, n_2) e^{-j(\frac{2\pi}{N}n_2 + \omega(\mathbf{n})\delta_2)m_2}.$$

3. Estimated image is $f(\mathbf{n}) = \frac{\tilde{f}(\mathbf{n})}{(-1)^{n_1}(-1)^{n_2}}$.

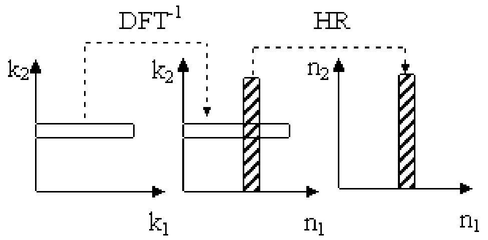


Fig. 2. Illustration of the proposed approach.

3.4. Condition on the field map smoothness

The HR problem (9) solves for each of the columns of the image $f(\mathbf{n})$ and corresponding term $\phi_n = \omega(\mathbf{n})\delta_2 + \frac{2\pi}{N}(n_2 + 1)\Delta_2$, but the order of ϕ_n (and hence $f(\mathbf{n})$) in general can not be preserved, since reordering of $f(n)$ and ϕ_n does not change the data $s(\mathbf{m})$. To deal with this, we need the function $\omega(n_1, n_2)\delta_2 + \frac{2\pi}{N}(n_2+1)\Delta_2$ to be monotonic, which requires the following constraint:

$$\begin{aligned} \omega(n_1, n_2)\delta_2 + \frac{2\pi}{N}n_2\Delta_2 \quad < \quad & \omega(n_1, n_2+1)\delta_2 + \\ & \frac{2\pi}{N}(n_2+1)\Delta_2, \end{aligned} \tag{11}$$

which is equivalent to

$$\omega(n_1, n_2) - \omega(n_1, n_2+1) \quad < \quad \frac{2\pi\Delta_2}{N\delta_2}, \quad \forall n_1, n_2. \tag{12}$$

4. EXPERIMENTAL RESULTS

In our experiments, a real MRI image and field map were acquired in accordance with the Internal Review Board (IRB) of Illinois. The MRI data is produced according to the following parameters: $\Delta_1 = \Delta_2 = 1$, $\delta_1 = 5\mu s$, $\delta_2 = 400\mu s$. We are interested in the SNR of the reconstructed image in two cases: with and without compensating for the field map. We also consider field maps satisfying and not satisfying the monotonicity condition (12). In the experiments shown below HR part was implemented using Forward-Backward method [5].

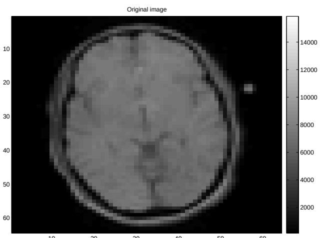
In the first case, field maps in the range from 15 Hz to 60 Hz were considered. In Fig.3 one of these experiments is shown: the original field map has a maximum value of about 27 Hz. The results show that our approach recovers the original image and field map much better (SNR=47.35 dB) compared to the case where field map is not considered (SNR=5.03 dB).

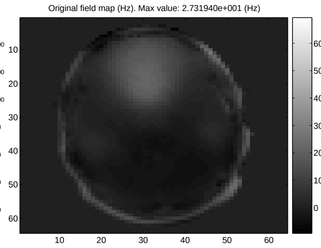
In the second case, the original field map is more severe. It has a maximum value of 95 Hz (Fig. 4). In this case, due to the violation of the monotonic condition, at *a few* locations HR cannot recover the correct order of ϕ_n, thus causing pixel shifts and gaps in the reconstructed images. We further postprocess the images to keep track of the gaps and then shifts back all pixels. The resulting SNR is 18.62 dB, while the reconstruction without the field map gives SNR=2.38 dB. We can see that reconstructed images without field map are greatly distorted in the region of field inhomogeneity (Fig. 4(d)). Note also that with the increasing range of field inhomogeneity, the approximation $\omega(\mathbf{n})\delta_1 \approx 0$ is less preserved.

Another advantage of this method is running time. By making the approximation $w(\mathbf{n})\delta_1 \simeq 0$, we map the original 2D problem into a sequence of 1D HR problems, which reduces execution time dramatically compared to the CG method [3]. The program was run on an Intel Pentium 1.6 GHz CPU, 1.25 GB RAM computer. For a 64×64 image, the running time is about 13 sec. With a 90×90 image, the running time is about 43 sec.

5. CONCLUSION

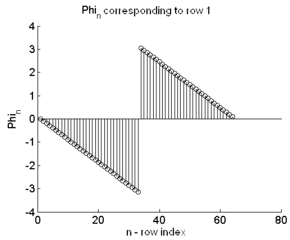
In this work, we propose a new method for the MRI Joint Estimation problem by making a practical approximation that gives us two advantages. First, the nonlinear Joint Estimation problem is transformed to a linear problem. Second, this approximation helps us to convert from a 2D Joint Estimation problem into a 1D HR problem. To make a bridge between the Joint Estimation and HR problems, we need a condition on the smoothness along vertical direction of the original field map or its variation range. Results show that image reconstruction indeed relies on good estimation of the field map, and a slight change in the field map can lead to large distortion of the image. With small and medium variation of the

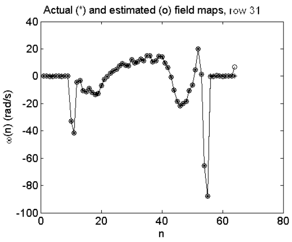




a) Original 64×64 image.

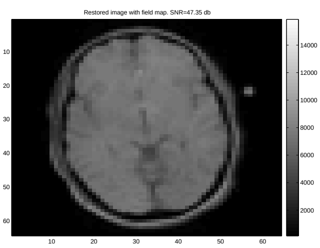
b) Original field map, max. value: 27.31 Hz.

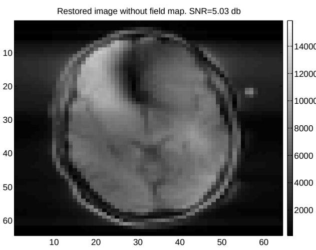




c) Monotonicity condition is held, row 1.

d) Actual (*) and estimated (o) field maps, row 31.

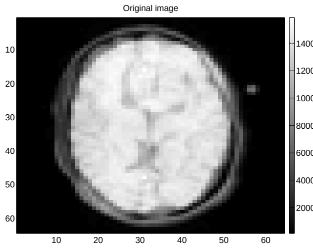


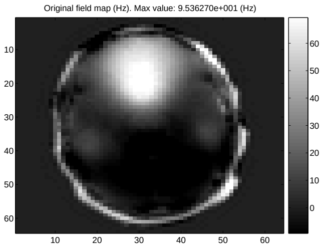


e) Recovered image using field map, SNR=57.95 dB.

f) Recovered image without field map, SNR=5.03 dB.

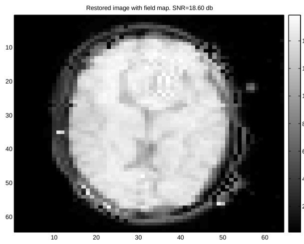
Fig. 3. Experiment with moderate field map.

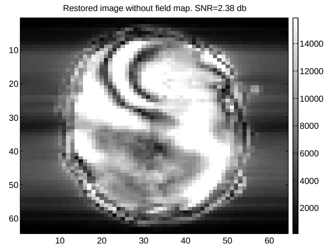




a) Original 64×64 image.

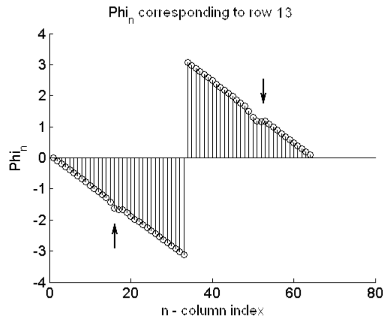
b) Original field map, max. value: 95.36 Hz.





c) Recovered image using field map, SNR=18.62 dB.

d) Recovered image without field map, SNR=2.38 dB.



e) Monotonicity is not held, row 13.

Fig. 4. Experiment with severe field map.

field map, our approach recovers the image much better in comparison to the method without the field map. In the case of the severe field map, we can still reconstruct all details and shape of the image, with slight pixel shifts. The main advantage of this approach is that it does not require a prior knowledge of the field map and has low complexity. The resulting estimated field map can also be used as an initial guess for other Joint Estimation methods [3].

6. REFERENCES

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